

Non-analytic Vortex Core and a Nonlinear Vortex Flow in Bosonic Superfluids

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We analyze the disorder limited motion of quantum vortices in a two-dimensional bosonic superfluid with a large healing length. It is shown that the excitations of low-energy degrees of freedom associated with the non-analytic reconstruction of the vortex core [Ann. Phys. **346**, 195 (2014)] determine strong non-linear effects in the vortex transport at velocities much smaller than Landau's critical velocity. Experiments are suggested to verify our predictions.

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Introduction– Entropy production and dissipation in superfluids and superconductors are associated with the dynamics of vortices. The cores of these vortices play a central role in these processes. For instance, the Bardeen-Stephen friction force acting on a vortex in type II superconductors comes essentially from the current flowing through its normal core [1]. Beyond the linear response regime, Larkin and Ovchinnikov showed that the quasi-particle distribution within the vortex core deviates from the equilibrium distribution due to the long inelastic relaxation time. The heating in the core of the vortex leads to a nonlinear behavior of the friction force acting on the vortex even when the value of the applied current is much smaller than the critical one [2–4].

There is no analogous scenario for vortex flow in bosonic superfluids. This is because, unlike superconductors (and ³He fermionic superfluid), quantum vortices in such a superfluid were thought to possess an essentially featureless cores, see e.g. Ref. [5].

However, we showed recently [6] that vortices in two-dimensional bosonic superfluids experience non-analytic reconstruction of their cores when moving with respect to the flow. The theory [6] developed for the limit, $n\xi^2 \gg 1$, where n is the bosonic density and ξ is the healing length (i.e. the size of the vortex core), predicts the following: i) The low energy degrees of freedom are not exhausted by the position of the vortex itself but must include the precession of the vortex around its guiding center; ii) This precession can be characterized by the kinetic momenta $\hat{\vec{p}} = (\hat{p}_x, \hat{p}_y)$ such that

$$[\hat{p}_x, \hat{p}_y] = i2\pi\sigma n\hbar^2, \quad (1a)$$

where n is the bosonic density far from the vortex core, and $\sigma = \pm 1$ is the vorticity, Eq. (1a) corresponds to the Lorentz (Magnus) force acting on a moving quantum vortex with respect to the superfluid; iii) The momentum dependence of the kinetic energy is non-analytic

$$\hat{H}_k(\hat{\vec{p}}) = \frac{\hat{p}^2}{2M_v(\hat{p}^2)}; \quad \frac{M_v(p^2)}{mn\xi^2} \equiv \frac{\pi}{\alpha^2} \ln \left(\frac{\hbar^2 n^2 \xi^2}{p^2} \right), \quad (1b)$$

where m is the boson mass, and $\alpha = 0.802 \dots$; iv) Semi-

classical quantization $\hat{H}_k(\hat{\vec{p}})$ gives discrete energy levels

$$\epsilon_{l+1} - \epsilon_l = \hbar\omega_c^l; \quad \omega_c^l = \frac{2\alpha^2\hbar}{m\xi^2} \left[\ln \left(\frac{\hbar\omega_c^l n \xi^2}{\epsilon_l} \right) \right]^{-1}, \quad (1c)$$

where $l \geq 0$ is an integer; v) Excited states ($l \geq 1$) decay due to the phonon emission but the relaxation time, τ_{in} , is large and discrete levels are distinguishable

$$\frac{1}{\tau_{in}^l} = \frac{2\pi\alpha^8\hbar}{m\xi^2} \left[\ln \left(\frac{\hbar\omega_c^l n \xi^2}{\epsilon_l} \right) \right]^{-4}. \quad (1d)$$

The spectrum (1b) with vortex mass $M_v \simeq mn\xi^2$ was discussed extensively in the literature concluding that the excited states of the vortex are not relevant for the low-energy dynamics. The large logarithmic factor in the mass, see Eq. (1b), makes the dynamics much slower than it was previously thought [6].

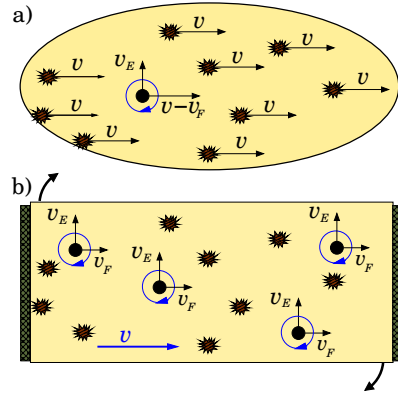


FIG. 1. Setups for a) cold atom (BEC) based and b) helium films superfluids. In BECs, vortex (black dot) can be introduced by phase imprinting [7], while motion is induced by sweeping (with velocity v) a disordered potential generated by a speckle pattern (brown impurities) [8]. For helium films vortices can be introduced by rotating the sample [5] (so that the vortex lattice is not yet formed), and the superfluid motion is obtained by evaporative heater located at one end of the system and a reservoir of superfluid helium at the other end.

Whereas a direct observation of the core dynamics is difficult, its manifestation via the dissipative motion of

vortices, like those in superconductors [2, 3], is experimentally accessible, as we demonstrate in this Letter.

In a stationary clean superfluid the vortices move with the flow and the internal degrees of freedom are not excited. The experiments to pinpoint the excitations in disordered systems are sketched in Fig. 1. For a BEC [Fig. 1a)] the motion of the vortices can be observed *in situ* whereas for the helium film realization [Fig. 1b)] it can be deduced from measurements of the chemical potential gradient along the superfluid flow by means of differential pressure transducer [9]. The superfluid flow is obtained by evaporative heater located at one end of the system and a reservoir of superfluid helium at the other end [10]. Vortices with the same vorticity are induced by mounting the sample in a rotating cryostat, see e.g. Ref. [11].

The main quantities are the non-linear susceptibilities:

$$v_F = \chi_F(v)v; \quad v_E = \sigma\chi_E(v)v, \quad (2)$$

where the vortex, $v_{E,F}$, and superfluid velocities, v , are introduced in Fig. 1 (v 's are to be understood as averages over the disorder realizations). Our predictions for the susceptibilities are summarized in Fig. 2 and the analytic expressions are given at the end of this paper.

In BEC systems, the motion of the vortex can be imaged and $\chi_{F,E}$ are directly measurable. For helium films the chemical potential gradient, $\vec{\nabla}\mu$, satisfies the relation

$$\vec{\nabla}\mu = -2\pi\hbar N_V \chi_E(v)v, \quad (3)$$

where N_V is the density of the vortices per unit area. Thus $\chi_E(v)$ can be extracted from the ratio of the chemical potential gradient to the superfluid velocity. Moreover, the N-shape of $\chi_E(v)v$ leads to the situation where the same vortex current can be realized for three possible value of the supercurrent v , thus leading to the filament instability of the supercurrent itself in large samples.

Qualitative discussion – To discuss the motion of the vortex it is necessary to supplement the kinetic energy (1b) with the fields coming from the motion of the superfluid surrounding the vortex. The corresponding effective theory [6] is conveniently written within Popov's formalism [12] mapping the problem of two-dimensional superfluid to two-dimensional nonlinear electrodynamics. In this mapping, vortices become charged particles with charge $\sigma 2\pi\hbar$, the electric field E is related to the superfluid current \vec{j} as $\vec{E} = -\hat{\epsilon}\vec{j}$ (where $\hat{\epsilon}$ is the antisymmetric tensor of the second rank acting on the spatial coordinates), and the magnetic field, B , is the boson density, n . In Popov's variables, the effective Hamiltonian of a vortex (which includes the reconstruction of the core) is

$$\hat{\mathcal{H}} = H_k(\vec{p}) + 2\pi\sigma\hbar\varphi + \frac{\vec{p}\hat{\epsilon}\vec{E}}{B} + \frac{\pi\hbar^2 B(\vec{r})}{m}, \quad (4)$$

where $\vec{p} = \vec{\mathcal{P}} - 2\pi\sigma\hbar\vec{A}(\vec{r},t)$ is the kinetic momentum of the vortex (1a), while $\vec{\mathcal{P}}$ is the canonical one, $\varphi(\vec{r},t)$

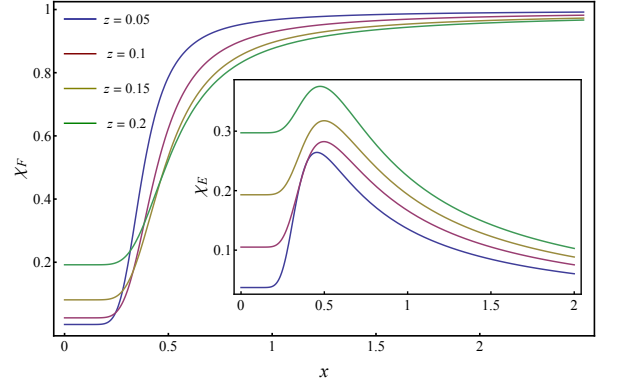


FIG. 2. Non-linear susceptibilities as the function of the velocity v expressed in terms of the “threshold” velocity v^* determined by the disorder and the phonon temperature $x \equiv v/v^*$. Different curves correspond to different phonon temperature T expressed in units of the disorder energy ϵ_d , see Eq. (7a), $z = T/\epsilon_d$. Naturally, the threshold behavior becomes more pronounced with the lowering of the temperature.

and $\vec{A}(\vec{r},t)$ are the scalar and the vector potentials, respectively. The physical fields are $\vec{E} = -\vec{\nabla}\varphi - \partial_t\vec{A}$ and $B = \vec{\nabla} \times \vec{A}$. The gauge invariance of Eq. (4) is nothing but vorticity conservation.

The third and the fourth terms in Eq. (4) are not present in usual electrodynamics. The third term expresses the fact the vortex executes its motion around the guiding center moving together with the superfluid and manifests Galilean invariance. The last term is the energy of the core depending on the superfluid density outside the core, $B(\vec{r})$. We will see shortly that this term is important for the scattering of the vortex by disorder.

It is instructive to write the equation of motion from the Hamiltonian (4). Suppressing the non-important effects of the spatial inhomogeneity of E/B , we have

$$\dot{\vec{p}} = 2\pi\sigma\hbar \left(\vec{E} + B\hat{\epsilon}\vec{r} \right) - \pi\hbar^2 \vec{\nabla} B(\vec{r})/m \quad (5a)$$

$$\dot{\vec{r}} = \partial_{\vec{p}} \hat{H}_k(p) + \hat{\epsilon}\vec{E}/B. \quad (5b)$$

and combining Eqs. (5a)–(5b), we find

$$\dot{\vec{p}} = \pm\omega_c(p^2)\hat{\epsilon}\vec{p} - \pi\hbar^2 \vec{\nabla} B(\vec{r})/m, \quad (5c)$$

i.e. the electric field cannot excite the external degrees of freedom of the vortex. Such excitations can be induced only by scattering on inhomogeneities of the bosonic density that we will describe now.

The analogy of the problem with the motion of electron in magnetic field enables us to use the formalism developed for the nonlinear magnetotransport in a two dimensional electron gas [14]. Consider the small Gaussian variations in the boson density $n = n_0 + \delta n$,

$$\langle \delta n(\vec{r}_1) \delta n(\vec{r}_2) \rangle_q = \gamma n_0 G(qr_c), \quad (6)$$

where $\langle \dots \rangle_q$ denotes the disorder averaging and Fourier transform over $\vec{r}_1 - \vec{r}_2$, the parameter $\gamma \ll 1$ describes the disorder strength, r_c is the correlation radius [13], and $G(x)$ is the dimensionless function whose precise form is not important for us provided that it drops fast enough at $x \gg 1$. From Eq. (1a) it follows that $p^2 \geq \hbar^2 n_0 \gg \hbar^2 / r_c^2$ which means that the vortex experiences small angle scattering by the disorder. The relaxation times can be estimated by treating the last term in Eq. (4) within the Fermi golden rule and neglecting the curving of the trajectories between scattering events. It gives the following relaxation time for the momentum direction:

$$1/\tau_{tr}(\epsilon) = \omega_c (\epsilon_d/\epsilon)^{3/2}, \quad (7a)$$

where ϵ_d is the characteristic energy scale [15] associated with the disorder. If the kinetic energy of the vortex ϵ is larger than ϵ_d the vortex precesses many times before changing its position, otherwise the vortex scatters into the new position before it manages to complete the circle.

There is another time scale describing the scattering at all angles (and not only those which change the direction of the momentum significantly) [15]:

$$1/\tau_q(\epsilon) = \omega_c (\epsilon_q/\epsilon)^{1/2}, \quad \epsilon_q \gg \epsilon_d. \quad (7b)$$

If $\omega_c \tau_q \gtrsim 1$, one can neglect the interference associated with the coming back to the same scattering center (Shubnikov-de-Haas effect). We will assume $\epsilon < \epsilon_q$, whereas the relation between ϵ_d and ϵ may be arbitrary.

Consider now a vortex in the coordinate frame moving with velocity $\vec{v} = \hat{e} \vec{E}/B$ [moving disorder in this frame does not change H_k because of the last term in Eq. (4)]. To be at rest in the laboratory frame (which would be consistent with disorder pinning), a vortex should have the directed velocity $-\vec{v}$. If there were no disorder, $\tau_{tr} \rightarrow \infty$, this directed velocity would precess and average to zero. The presence of disorder allows for rotation by small angle $\omega_c \tau_{tr} \ll 1$. As a result the velocity acquires a component along E , $v_E \simeq \sigma \omega_c \tau_{tr} \vec{E}/B$. The motion along the electric field leads to Joule heating, and the power produced by vortices with typical energy $\epsilon < \epsilon_d$ [see Eq. (7a)] is

$$P(\epsilon)/(2\pi\hbar) \simeq \sigma v_E E \simeq \omega_c \tau_{tr} \vec{E}^2/B \simeq \vec{E}^2/B (\epsilon/\epsilon_d)^{3/2}$$

In the opposite limit $\omega_c \tau_{tr} \gg 1$, the circular motion averages out the dissipative current, and only rare scattering events contribute to the dissipation power. Therefore the dissipative current should be proportional to $1/\tau_{tr}$, and on dimensionality grounds it leads the replacement, $\omega_c \tau_{tr} \rightarrow 1/(\omega_c \tau_{tr})$, i.e. for $\epsilon \geq \epsilon_d$,

$$P(\epsilon)/(2\pi\hbar) \simeq \omega_c \tau_{tr} \vec{E}^2/B \simeq \vec{E}^2/B (\epsilon_d/\epsilon)^{3/2},$$

Thus the generation of energy is a peaked function of ϵ , and non-equilibrium effects are associated with its

particular form. If there were no inelastic processes the distribution function of the vortices in the energy space $f(\epsilon)$ would never be stationary. Phonon emissions (1c) remove the energy from the vortex core, and the extra energy accumulated by the vortex with reference to the starting energy ϵ can be estimated as

$$\Delta(\epsilon) \simeq (2\pi\hbar\tau_{in}E^2/B) \left[(\epsilon_d\epsilon)^{3/2} / (\epsilon_d^3 + \epsilon^3) \right]. \quad (8)$$

Therefore, for large enough E , there exists a region where $\Delta(\epsilon) \gtrsim \epsilon$. The distribution function in such a region is almost constant, see Fig. 3. Non-equilibrium currents, however, are determined by the energy derivative of the distribution function $(-\partial f/\partial\epsilon)$, shown in inset of Fig. 3. Thus, the currents are not determined by the whole distribution function but only by regions at small and large energies where the dissipative currents are suppressed. This explains the non-linearity of dissipation as function of E/B , and the drop in the dissipative current at large E/B . The quantitative qualitative requires full kinetic description of the problem outlined below.

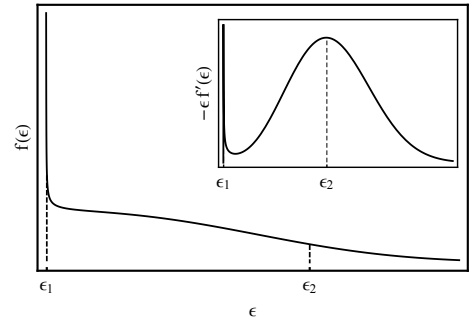


FIG. 3. The shape of the non-equilibrium distribution function $f(\epsilon)$ calculated from Eq. (15). The steep drop at low energies is the thermal distribution and the high energy tail originates for the energy resolved “heating” (8). For $\epsilon_1 < \epsilon < \epsilon_2$, the relation $\Delta(\epsilon) > \epsilon$ holds.

The kinetic equation has the standard form in the energy-angle variables [16]. Suppressing the spatial dependence of the distribution function, $f(\epsilon, \phi)$, we obtain

$$\partial_t f + \sigma \omega_c(\epsilon) \partial_\phi f = St_{el}[f] + St_{in}[f]. \quad (9a)$$

The collision integrals in the right-hand-side of Eq. (9a) describe probabilistic processes. The disorder generates small angle scattering (angular diffusion):

$$St_{el}[f] = \left[\frac{\partial}{\partial\phi} - \frac{\partial}{\partial\epsilon} \frac{\vec{E} \cdot \vec{p}}{B} \right] \frac{1}{\tau_{tr}} \left[\frac{\partial}{\partial\phi} - \frac{\vec{E} \cdot \vec{p}}{B} \frac{\partial}{\partial\epsilon} \right] f, \quad (9b)$$

where the elastic relaxation time $\tau_{tr}(\epsilon)$ is given by Eq. (7a). We defined the angle ϕ so that $\vec{E} \cdot \vec{p} = Ep(\epsilon) \cos \phi$, and $p(\epsilon) = \sqrt{2\epsilon M_v(\epsilon)}$, see Eq. (1b). The extra term in addition to the angular derivative is the

Galilean correction to the vortex energy in the moving superfluid [third term in Eq. (4)]. The energy transfer in the phonon emission is small and can be described by Focker-Planck terms. Neglecting the effects of the field E and the disorder on the inelastic collision, we obtain

$$St_{in}[f] = \frac{\partial}{\partial \epsilon} \left[\frac{\epsilon}{\tau_{in}(\epsilon)} \left(1 + T \frac{\partial}{\partial \epsilon} \right) \right] f, \quad (9c)$$

where T is the phonon temperature, and the inelastic rate is given by Eq. (1c).

The vortex velocities (in the convention of Fig. 1) are

$$v_E = \int f \frac{\partial \epsilon}{\partial p} \cos \phi d\phi d\epsilon; \quad v_F = \frac{E}{B} + \int f \frac{\partial \epsilon}{\partial p} \sin \phi d\phi d\epsilon, \quad (10)$$

with the normalization $\int d\phi d\epsilon f = 1$.

Solution of the kinetic equation proceeds in a standard way *e.g.* considering the heating effects in metals. Let $f(\epsilon; \phi) = f_0(\epsilon) + f_1(\epsilon; \phi)$, and $\int d\phi f_1(\epsilon; \phi) = 0$. The angular dependent part of the distribution function f_1 is massive and can be found to first order in the perturbation in E . For the same reason, inelastic collision effects on f_1 can be also neglected, and we find

$$f_1 = \frac{Ep(\epsilon)}{B} \frac{\sigma \omega_c(\epsilon) \tau_{tr}(\epsilon) \cos \phi - \sin \phi}{1 + \omega_c(\epsilon)^2 \tau_{tr}(\epsilon)^2} \left(-\frac{\partial f_0}{\partial \epsilon} \right). \quad (11)$$

Substituting $f(\epsilon; \phi) = f_0(\epsilon) + f_1(\epsilon; \phi)$ back into Eq. (9a), using Eq. (11) and integrating the result over the angle ϕ we obtain the spectral diffusion equation $\partial_t f_0 + \partial_\epsilon j_\epsilon = 0$, where the spectral flow current is given by

$$j_\epsilon = -\epsilon f_0(\epsilon) / \tau_{in}(\epsilon) - D_\epsilon \partial_\epsilon f_0(\epsilon). \quad (12a)$$

The spectral diffusion is caused both by the inelastic processes and by Joule heating due to the electric field:

$$D_\epsilon = \frac{\epsilon T}{\tau_{in}(\epsilon)} + \frac{1}{2} \left(\frac{Ep(\epsilon)}{B} \right)^2 \frac{\omega_c(\epsilon)^2 \tau_{tr}(\epsilon)}{1 + \omega_c(\epsilon)^2 \tau_{tr}(\epsilon)^2}. \quad (12b)$$

In the stationary state the spectral flow is absent $j_\epsilon = 0$ and we obtain from Eqs. (12)

$$\frac{\partial f_0}{\partial \epsilon} + \frac{f_0}{T_{eff}(\epsilon)} = 0; \quad T_{eff} = T + \frac{2\pi \hbar \tau_{in} E^2}{B} \frac{(\epsilon_d \epsilon)^{3/2}}{\epsilon_d^3 + \epsilon^3}, \quad (13)$$

where we used $p^2 = 2M_v \epsilon$, $\omega_c = 2\pi \hbar B / M_v$, and the explicit energy dependence of the transport relaxation rate (7a). The meaning of the last term in the expression for the effective temperature T_{eff} has been already discussed in derivation of Eq. (8).

Equations (10) and (11) give

$$\begin{bmatrix} v_E \\ v_F \end{bmatrix} = \frac{E}{B} \int_0^\infty \left[\pm (\epsilon_d \epsilon)^{3/2} \right] \left(-\frac{\partial f_0}{\partial \epsilon} \right) \frac{\epsilon d\epsilon}{\epsilon_d^3 + \epsilon^3}. \quad (14)$$

The normalized solution of Eq. (13) is

$$f_0(\epsilon) = \frac{F(\epsilon)}{\int_0^\infty d\epsilon_1 F(\epsilon_1)}; \quad F = \exp \left[-\int_0^\epsilon \frac{d\epsilon_1}{T_{eff}(\epsilon_1)} \right]. \quad (15)$$

Substituting Eq. (15) into Eq. (14), restoring $v = E/B$, and matching overlapping asymptotes for the integrals we obtain for the susceptibilities $\chi_{\nu=E,F}$ of Eq. (2):

$$\chi_\nu = \frac{\left(\frac{2T}{\epsilon_d} \right)^{\delta_\nu^T} d_\nu^T + \left(\frac{v}{v_*} \sqrt{\frac{\epsilon_d}{T}} \right)^{\delta_\nu^v} e^{-(v^*/v)^{4/3}} d_\nu^v}{\left(\frac{T}{\epsilon_d} \right) + \left(\frac{v}{v_*} \sqrt{\frac{\epsilon_d}{T}} \right)^{\delta_F^v} e^{-(v^*/v)^{4/3}} d_F^v}, \quad (16a)$$

where the exponents are $\delta_E^v = -2/5$; $\delta_F^v = 4/5$; $\delta_E^T = 5/2$; $\delta_F^T = 4$, and the numerical prefactors are all of the order of unity: $d_E^T = 0.58 \dots$, $d_F^T = 3/2$, $d_E^v = 0.583 \dots$, $d_F^v = 3.53 \dots$. The nonlinearity occurs at “threshold” velocity

$$\frac{v_*}{c} = \left[\frac{\beta}{n_0 \xi^2} \frac{m}{M_v} \frac{\hbar \omega_c}{T} \right]^{1/4} \left[\frac{m \xi^2 \epsilon_d}{\hbar^2} \right]^{3/4} \left[\frac{1}{\omega_c \tau_{in}} \right]^{1/2}, \quad (16b)$$

where $\beta = 32\pi^2 / (81\sqrt{3}) = 2.25 \dots$, and $c = \hbar / (m\xi)$ is the speed of sound. With the logarithmic accuracy, one uses $\epsilon_l \rightarrow T \gtrsim \hbar \omega_c$ in expressions (1c)-(1d). At smaller temperatures, one should replace $T \rightarrow \hbar \omega_c (\epsilon_l = \hbar \omega_c)$. Each fraction in Eq. (16b) is small so that the non-linearity occurs at a superfluid velocity much smaller than Landau’s critical value. This value of v_* can be understood from the condition $T_{eff}(\epsilon) = \epsilon = 2T$, whose meaning is obvious from the qualitative discussion and Fig. 3.

In conclusion, we constructed the theory for the motion of quantum vortices in disordered two-dimensional bosonic superfluids. The excitations of low energy degrees of freedom, associated with core reconstruction [6], lead to non-linear transport phenomena, see Eqs. (16) and Fig. 2, resembling those in superconductors [2, 3]. The confirmation of the peak effect in the dissipation and the threshold behaviour in the drift provides evidence for the existence of the vortex core reconstruction and further our understanding of the dissipative vortex transport.

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- [13] It is natural to assume $r_c \gtrsim \xi$ as the weak disordered potential is linearly screened by the bosons which suppresses $q \gtrsim 1/\xi$. The additional reason for this assumption is the averaging of the density in the vortex core.
- [14] See e.g. M. G. Vavilov and I. L. Aleiner Phys. Rev. B **69**, 035303 (2004) for the systematic derivation of the kinetic equation.
- [15] The expressions for the energies $\epsilon_{d,q}$ in terms of the parameters of the model, see Eqs. (1) – (6) are

$$\begin{pmatrix} \epsilon_d \\ \epsilon_q \end{pmatrix} = \frac{\hbar^2}{2mr_c^2} \begin{pmatrix} \gamma^{2/3} \left(\frac{M_v}{m}\right)^{1/3} \left[\frac{\pi}{4} \int dx x^2 G(x)\right]^{2/3} \\ \gamma^2 \left(\frac{M_v}{m}\right)^3 \left[\frac{\pi}{2} \int dx G(x)\right]^2 \end{pmatrix}$$
- [16] We will not write terms for the elastic scatterings of phonons on the vortex which do not change our results qualitatively. We also omitt logarithmic energy dependence in the votex density of states. It implies that with logarithmic accuracy, $2\epsilon(p) \approx p\partial_p\epsilon(p)$, and all the logarithms in the integrals are treated as constants with the energy argument of the logarithm replaced with the relevant energy scale.